

TAP TONES AND WEIGHTS OF OLD ITALIAN VIOLIN TOPS

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Abstract

The tap tones of Old Italian violin tops and backs have long interested makers and researchers, both as clues to how the classical makers graduated their instruments, and as guides for graduating instruments today. The frequencies of the tap tones reflect the relationship between stiffness and mass in the plate. If the mass is also known, then one can work backward to determine the stiffness. In this article, the tap tones and weights of nine Old Italian violin tops are examined with a view to 1) assess their relative stiffness, 2) compare them with new tops, and 3) estimate the so-called radiation ratio of their wood, which would improve the choice of materials for either restoring or copying them.

Hold a violin top between two fingers and tap it, and one or more distinct pitches, or “tap tones,” can be heard. No one knows what the early Italian violinmakers made of tap tones, or if they used them in any systematic way during graduation—still, the tap tones of Old Italian violins have interested makers and researchers going back at least as far as Félix Savart in the early 19th century [1]. In his book *The Art of Violin Making*, the German maker Otto Möckel [2] compiled a record in which he rendered tap tones in terms of musical notation; the pitch of the top of a 1712 Stradivari violin, for example, is listed as “f+1/4 tone.” Beginning in the early 1960s Carleen Hutchins [3, 4] measured the tap tones of a Stradivari top, found an octave relationship between modes 2 and 5, and developed an influential method for free plate tuning. She and others also developed the sine-wave-and-glitter technique for both reading the frequency of tap tones and making visible their underlying vibrational modes, or “Chladni patterns.”

In 1986 Gregg Alf and I invited Hutchins to teach a work-

shop on plate tuning at our shop. Since then I have tuned all my plates, though what I mean by plate tuning has shifted over time to mean something like “using tap tones as a guide to intelligent graduation.” Some years ago I switched from sine waves and glitter to a computer-based method for reading tap tones using a sound analysis program called SpectraPlus [5]. The plate is held in front of a microphone, tapped in the usual fashion, and with a couple of cursor movements, the frequencies of the tap tones can be read off the monitor.

It is one of the oddities of our profession that although we have detailed records of almost every aspect of Old Italian violins, from graduation patterns to purfling points, there is almost nothing available on how much a typical Old Italian top or back weighs. I never thought of weighing my own plates until Carleen Hutchins suggested it in 1986. Over the years I have weighed the occasional Old Italian violin that came my way, and more recently have begun to assemble a database of weights and tap tones for historical instruments. This article focuses on the tops of nine Old Italian violins, all of them concert instruments with several of them in the hands of major soloists, including Elmar Oliveira, Vladimir Spivakov, and Maxim Vengerov. Some are instruments that Gregg Alf and I copied. The others were from the collections of Florian Leonard and J.&A. Beare.

There is a fair amount of literature devoted to tap tones and plate tuning. In this article I assume that the reader is familiar with the basic concepts. My interest is not in proposing a system for tuning plates, nor in speculating about the workshop practice of the early Italian violinmakers. Rather, I am interested in what weight *in combination with* two particular tap tones of modes 2 and 5 can tell us about 1) the relative stiffness of Old Italian violin tops, 2) the mechanical properties of the wood used, and 3) the sort of wood needed to build a new top that matches an old one in terms of stiffness and weight.

Tap Tones and Plate Modes

Given the top and back of a great Old Italian violin, I believe that an experienced violinmaker could build a complete instrument around these parts, and provided the structure and setup of the original were respected, the “new” instrument would sound much like the original. My underlying assumption is that the vibratory characteristics of the top and back are the main determinants of an instrument’s sound. It happens that the vibratory characteristics of a violin plate or any other object can be completely described in terms of a set of the normal modes of vibration, also known as “eigenmodes.” Each mode is characterized by a shape, characteristic value(s) of frequency, damping parameter, and

impedance. By convention, the modes are numbered in ascending order, beginning with the lowest in frequency: mode 1. The shape and frequency of each mode are determined by the interplay between stiffness and mass in the structure. Stiffness and mass are, in turn, determined by the geometry of the structure and the mechanical properties of its constituent materials.

The term “tap tone” usually refers to one or another of the plate’s lowest normal modes, typically modes 1, 2, and 5. These three are “signature” modes in that they reflect, more purely than any other modes, the torsional, lateral, and longitudinal stiffness of the plate. Plate stiffness is determined by many factors, including outline, arching, and graduation. Once these are taken into account, the stiffness of the plate is a reflection of the stiffness of the wood. Modes 1, 2, and 5 for the top (without a bass bar) can therefore be related to the three most important stiffness moduli of the wood: torsional stiffness (resistance to twisting), stiffness across the grain, and stiffness along the grain. Mode 1 is strongly determined by torsional stiffness, while modes 2 and 5 are somewhat less strongly determined by stiffness across and along the grain respectively (more on this later). The frequencies of all other plate modes are determined by a more complex interaction of these moduli, and are therefore less interesting for our purposes.

Tap Tones, Plate Stiffness, and Graduation

Makers typically assess stiffness by flexing a plate in various directions. While the consistency and accuracy that comes with experience and collective know-how should never be underestimated, this kind of subjective judgment is difficult to quantify and thus to record for future reference. To get around this, some makers have built devices using weights to load the plate and micrometers to measure the resulting deflection. I believe that tap tones and weight provide much the same information, and they are also more meaningfully shared with colleagues who do not have identical deflection devices. The important thing to remember is that stiffness can be determined in several ways. In principle, cross-referencing any two of deflection (under standard loading conditions), mode frequency, and mass should work. If we know the frequency of the tap tones, we know something about the plate’s stiffness-to-mass ratios. If we also know the mass, we can work backward to obtain the stiffness. Thus, tap tones and weight together form a kind of shorthand for recording and tracking changes in plate stiffness.

In practice, there is no easy way to get from tap tones and weights to absolute stiffness values, though I will later suggest some guidelines for estimating relative values. I should say that while graduating my own instruments I am comfortable staying

in the realm of tap tones. I record the frequency of modes 2 and 5 along with the weight and let the stiffness take care of itself.

When a plate is thinned, its stiffness drops far more quickly than its mass. This is because the mass drops linearly, while the stiffness drops with the cube of the thickness. Therefore, changing the thickness of a plate *always* changes its stiffness-to-mass ratios. (Note: There are many possible stiffness-to-mass ratios for an instrument, depending on the direction of measurement, and these ratios vary locally with changes in wood stiffness, thickness, arching, etc.) The stiffness-to-mass ratios determine the speed at which bending waves move through a plate, and this is of primary importance to the overall acoustical behavior.

Imagine two tops with identical stiffness-to-mass ratios, but one weighing twice as much as the other. All else being equal, they would have identical mode frequencies. The heavier top would feel stiffer when flexed; it would also have twice the impedance—that is, a given force would produce half the amplitude of vibration. It is also true that the absolute mass of the plate has an importance that is independent of its stiffness-to-mass ratio. In very general terms, a high stiffness-to-mass ratio leads to efficient high-frequency radiation, while low absolute mass helps with power and response.

Tap Tones and Weights for Tops of Old Italian Violins

All tap tones were read using digital measuring equipment, so I am confident that they are accurate to within a few Hertz. Tap tones do vary somewhat with humidity, as do plate weights, which were measured to an accuracy of plus or minus a few tenths of a gram.

Table 1 shows data for nine Old Italian violin tops. M2 and M5 refer to modes 2 and 5, respectively. The first three columns of data are for tops without a bass bar, the next three for tops with a bass bar, and the final column is the weight of the bass bar itself. Where there are blank spaces, the instrument was not available for measuring both with and without bass bar. A summary of the data listed in Table 1 is:

- Without a bass bar, mode 2 ranged from 117 to 143 Hz; the average for the nine violins was 131 Hz.
- With a bass bar, mode 2 ranged from 276 to 322 Hz; the average was 309 Hz.
- Without a bass bar, top weights ranged from 54 to 65.5 gm; the average was 59.9 gm.
- With a bass bar, top weights ranged from 58 to 68.4 gm.

- The weight of the bass bars ranged from 4 to 4.5 gm; the average value was 4.3 gm.

Table 1. Tap tone frequencies and weight for the tops of nine Old Italian violins, with and without bass bar.

Instrument	M 2 (Hz)	M5 (Hz)	Wt. (g)	M2 (Hz)	M5 Hz	Wt. (g)	Bass Bar (g)
<i>Booth Stradivari (1716)</i>	127	305	54	150	345	58	4
<i>Kreutzer Stradivari (1727)</i>	117	276	55.5	139	324	60	4.5
<i>Petri Stradivari (1700)</i>	126	332	65.5				
<i>Artot-Alard Stradivari (1728)</i>				146	351	66	
<i>Stretton Guarneri(1726)</i>	143	308	64.1	155	362	68.4	.3
<i>Landolfi</i>				172	371	63.5	
<i>Tononi</i>				146	384	67.2	
<i>Testore (Spivakov)</i>	143	322	60.5	164	366	65	4.5
<i>Ruggieri, il Per</i>				171	375	65.5	
Average	131	309	59.9	155	360	64.2	4.3

Table 2 is an updated version of Table 1 with the blank spaces replaced by 1) taking the average bass bar weight, and then subtracting it from the total weight to get an estimated weight-without-bar, and 2) calculating the average percent frequency shift produced by the bar, and then “virtually” removing the bar. This was done mainly to see how the averages changed when these additional instruments are included. In fact, the averages go up slightly, as can be seen by comparing the bottom two rows in Table 2. Statistically, nine violins provide a small database from which to draw reliable conclusions. Still, whether the average weight for Old Italian violin tops is closer to 59.9 gm or 60.4 gm, my guess is that it is less than the weight of most new tops.

Table 2. Tap tones and weights of nine Old Italian violins.

Instrument	M 2 (Hz)	M5 (Hz)	Wt. (g)	M2 (Hz)	M5 Hz	Wt. (g)	Bass Bar (g)
Booth Strad, (1716)	127	305	54	150	345	58	4
Kreutzer Strad (1727)	117	276	55.5	139	324	60	4.5
Petri Strad (1700)	126	332	65.5				
Artot-Alard Strad (1728)	127	304	61.7	146	351	66	4.3
Stretton Guarneri (1726)	143	308	64.1	155	362	68.4	4.3
Landolfi	150	321	59.2	172	371	63.5	4.3
Tononi	127	332	62.9	146	384	67.2	4.3
Testore Spivakov	143	322	60.5	164	366	65	4.5
Ruggieri, <i>il Per</i>	150	324	61.2	171	375	65.5	
Average	134	314	60.4	155	360	64.2	
Previous average	131	309	59.9	155	360	64.2	4.3

*Shading indicates estimated values.

What do all these data imply about the relative stiffness of the tops? For mechanical resonances in general, stiffness is proportional to mass: If the mass increases 10%, then so must the stiffness if the frequency is to remain constant. With this in mind, consider the tops (without bass bar) of the *Booth* Stradivari and the *Lady Stretton* Guarneri *del Gesù*. The frequency of mode 5 is nearly identical for each, but the *Stretton* is roughly 20% heavier than the *Booth*. We can therefore conclude that the stiffness governing mode 5 (which I will refer to as “mode 5 stiffness”—more on this later) is about 20% greater for the *Stretton* than for the *Booth*.

Now consider the *Booth* and *Kreutzer* Stradivari violins (again without a bass bar). For the sake of simplicity, let’s say that their weights are identical. (In fact, the *Kreutzer* top plate is 1.5 grams heavier). The frequency of mode 5 for the *Kreutzer* Strad is ~10% lower. It turns out that for mechanical resonances, stiffness is proportional to the *square* of the frequency, as long as mass is kept constant. This means that a relatively small difference in frequency indicates a relatively large difference in stiffness. Mathematically, it translates into a rather convenient rule-of-thumb: a difference in frequency of a given percentage will indicate a difference of stiffness of *twice that percentage*—again

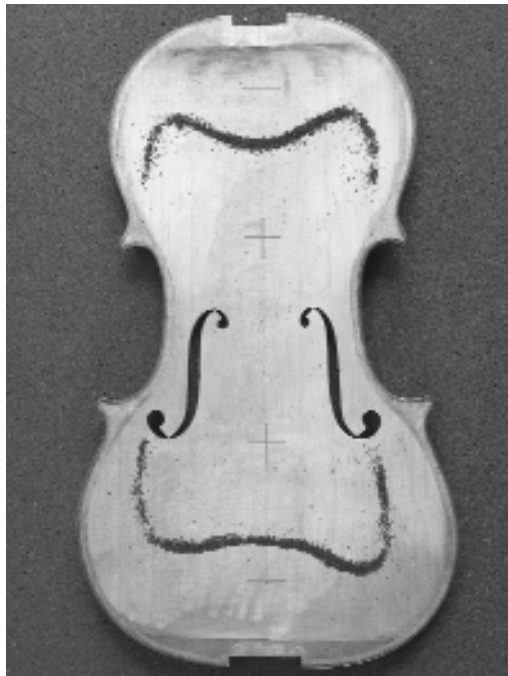
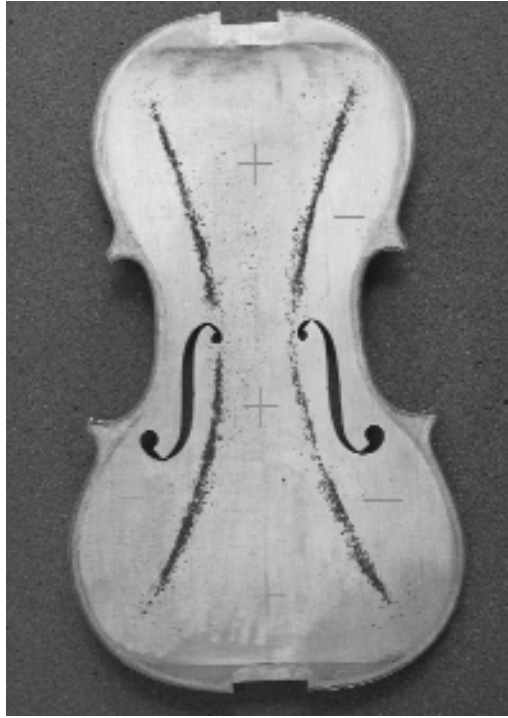
assuming that mass is constant. Since mode 5 for the *Kreutzer* is ~10% lower than for the *Booth*, then mode-5 stiffness is ~20% lower for the *Kreutzer*. Estimates for the mode-2 stiffness can be made in the same way.

What do I mean, exactly, by the terms “mode-2 stiffness” or “mode-5 stiffness”? Figure 1 shows modes 2 and 5 for a typical violin top. The + and – areas vibrate in opposite phase to each other—one area bending outward while the other bends in. These bending areas “pivot” about the nodal lines—lines connecting points of zero motion. There is by definition no bending at the nodal lines of a mode during vibration (at least when that mode *alone* is excited), but an increasing amount of bending as one moves away from the nodal lines. For this reason, the stiffness of a plate along the nodal lines is relatively unimportant to mode frequency, but the stiffness becomes increasingly important as one moves away from the lines. (Practically speaking, removing wood along a nodal line will barely affect the frequency of the mode, while removing wood far from a nodal line will tend to have a pronounced effect.) Thus, the stiffness governing a mode, for which I use the terms “mode-2 stiffness” and “mode-5 stiffness,” refers in very general terms to the stiffness of the *bending* areas of the plate.

Tap Tones and Wood Properties

Let’s consider two hypothetical tops, one a dimensionally exact copy of the other. Because their geometries are identical, any differences in stiffness, mass, and tap tones must stem from differences in their wood. Looking again at Fig. 1, if the nodal lines for mode 2 were straight and ran parallel with each other, then we could assume that cross-grain stiffness was the only wood modulus that affected mode 2. Similarly, if the nodal lines for mode 5 were straight and parallel, then long-grain stiffness alone would be involved. The extent to which the nodal lines are curved indicates the extent to which both long- and cross-grain stiffness are involved. Thus, mode 2 reflects mainly the cross-grain stiffness of the wood, while mode 5 reflects the long-grain stiffness *plus* a significant amount of cross-grain stiffness.

The Old Italian violin tops in Table 1 are not, of course, dimensionally exact replicas of one another. They all differ somewhat in terms of outline, arching, and graduation, and therefore in the distribution of stiffness and mass. Still, I am struck by the apparent range of stiffness values that I measured for these valuable concert instruments. For example, by the above estimates the mode-5 stiffness of the *Booth* Stradivari is about 20% stiffer than that of the *Kreutzer* Stradivari. The *Lady Stretton* Guarneri is ~20% stiffer still.



Figures 1. Modes 2 (upper) and 5 (lower) for a typical violin top without a bass bar. The “+” and “-” areas vibrate in opposite phase to each other—one area bending outward while the other bends in. These bending areas “pivot” about the nodal line, which connect the points of zero motion.

Ratio of Mode 2 to Mode 5 for Old and New Violins

If I look at the tap tones of my last 30 tops (without a bass bar) and calculate an average for their frequencies, I get 155 Hz and 308 Hz for modes 2 and 5, respectively. While frequency of my mode 5 average comes very close to the 309 Hz average for this set of Old Italian tops, my mode 2 average is significantly higher—155 Hz compared with 131 Hz. I find it striking that mode 2 on almost all my tops is higher than even the highest of these Old Italian violin tops. Is this typical for new tops? I don't know, but I am eager to compare notes with other violinmakers regarding this. However, several possible reasons for why this is true for my own work come to mind:

1. *Choice of wood:* This seems to me unlikely: I have used both European and American spruce from a wide variety of logs and have never attempted to choose wood that was relatively stiff across the grain.

2. *My graduation patterns favor a relatively high value for mode 2:* I use rather typical modern graduations: ~3 mm between the *f*-holes and decreasing to ~2.4 mm in the bouts. The thinner central region found in many Old Italian tops tends to favor a relatively low value for mode 2 in relation to mode 5.

3. *My archings are lower than those of the average Old Italian violin:* Professor Jim Woodhouse of Cambridge University (UK) [6] has stated that arching height strongly affects mode 5, but has little effect on mode 2. High arching, then, would tend to widen the gap between modes 2 and 5. The arching height of my tops is almost always between 15.5 and 16 mm. The values of most of the Old Italian violins that Gregg Alf and I copied were more like 17 or 18 mm, mainly due to long-term distortion. If this is indeed the explanation, it suggests that one needs to pay careful attention to arching height when trying to match the tap tones of an Old Italian violin top. It also suggests a way to compensate for wood that happens to be unusually strong or weak in one particular direction.

4. *Over time, the wood used by the early Italian violinmakers has become relatively stiffer along the grain—or weaker across it—than when it was when new:* This seems to me plausible, and research to test this hypothesis is currently underway.

Tap Tones, Weight, and Choice of Wood

Let's say you want to copy a particular Old Italian violin top—not just in terms of its shape and appearance, but also in terms of

tap tones and weight. What are the characteristics of the wood needed? Alternatively, let's say you are restoring an Old Italian violin top and need to replace significant amounts of wood, but without changing the stiffness or weight of the plate. How do you find repair wood that makes this possible? Do you compare the densities of the new and old woods, or do you somehow compare their stiffness?

It turns out to be a particular relationship between density and stiffness that is important—a relationship that is captured by a term known as the “radiation ratio.” The radiation ratio is an expression of something like “stiffness-per-unit-mass” for a material. Higher radiation ratios allow for the construction of lighter and/or stiffer structures. There are several ways to get at a material's radiation ratio; the most convenient for violinmakers is to divide the speed of sound (as measured by a Lucchi Elasticity Tester, a.k.a. Lucchi Meter [7]) by the density of the wood. Density can be calculated by dividing volume by weight, or by the flotation method. (This works well for wedges of constant cross-section. Float one end in a bucket of water and then mark the water line. Divide the wet length by the total length to get the density. For greater accuracy, dip each end and take the average of the wet lengths. Because the density of water is 1 gm/cm^3 , the resulting measurement will also be in gm/cm^3 .)

Table 3 lists the longitudinal radiation ratios alongside the densities of various wood samples I had around the workshop. There is clearly a statistical correlation between low density and high radiation ratio among the different species. This correlation holds for individual pieces of the same species, so if you don't have a Lucchi Meter, choosing the least dense woods will tend to yield those with the highest radiation ratios.

Table 3. Density and radiation ratio for various types of wood

Wood	Radiation Ratio	Density (g/cm^3)
Curly Maple (typical)	6 – 7.5	0.55 – 0.65
Willow (one sample)	10	0.5
Norway Spruce (typical)	12 – 15.5	0.37 – 0.44
Balsa (one sample)	26	0.2

When it comes to choosing wood, here are a few generalizations. Assuming that other relevant properties are equal:

1. If the radiation ratio of the new and old wood matches along and across the grain, then modes 2 and 5 *and* the weight can be matched.
2. If the radiation ratio of the new wood is lower than that for the old wood, then a) if the tap tones match, the new top will be heavier and stiffer than the old top, or b) if the weights match,

- the new top will have lower tap tones and be less stiff.
3. If the radiation ratio of the new wood matches, but its density is lower, then the tap tones and weight can be matched, but for this to happen the new top must be left thicker than the original.
 4. When considering replacement wood for restoration, the original stiffness and mass of a plate can be preserved only by finding wood with the same radiation ratio across *and* along the grains. To preserve the original graduations, the densities must also match.
 5. Run-out lowers the radiation ratio along the grain, and off-quarter grain orientation dramatically reduces the radiation ratio across the grain. This means that if the radiation ratio is good along the grain, but too high across it, a somewhat off-quarter cut might correct the ratio.

Assessing the Radiation Ratio of Old Italian Violin Tops

Although the radiation ratio is fairly straightforward to measure in chunks of new wood, it is by no means easy with a finished top. The density is difficult to determine without knowing the volume of the top, and the speed of sound as measured over the arching tends to be lower than when measured directly through the wood. Is there perhaps a way of determining the radiation ratio obliquely via tap tones and weight? Swedish researchers Molin, Lindgren, and Jansson [8] provide a method for estimating how an increase in plate thickness will raise the frequency of the tap tones. Through both experiment and computer modeling, they developed the following formulas:

- A 10% increase in top thickness will raise Mode 2 by ~7.3%.
- A 10% increase in top thickness will raise Mode 5 by ~5.5%.

Now, imagine that our nine Old Italian violin tops are laid out on the workbench. How much wood must be added or removed from each one in order for it to have the same mode 5, for example, as the *Booth* Stradivari? The second Swedish formula allows us to estimate this, in effect, to “virtually regraduate” the Old Italian violin tops until each mode 5 is tuned to 305 Hz. The results are shown in Table 4. The first two columns list the original mode 5 frequencies and weights. The next two columns show the newly calculated values. Note: the shaded rows are instruments for which there were only estimated values for the top without a bass bar; the resulting values are therefore even more speculative than the others.

Table 4. Mode 5 “normalized” to 305 Hz on all violin tops.*

Instrument	M5 (Hz)	Wt. (g)	M5_{norm} (Hz)	Wt._{norm} (g)
<i>Booth Strad</i> , 1716	305	54	305	54
<i>Kreutzer Strad</i> , 1727	276	55.5	305	65.1
<i>Petri Strad</i> , 1700	332	65.5	305	56
<i>Artot-Alard Strad</i> , 1728	304	61.7	305	62
<i>Stretton Guarneri</i> , 1726	308	64.1	305	63.8
<i>Landolfi</i>	321	59.2	305	53.8
<i>Tononi</i>	332	62.9	305	53.6
<i>Testore, Spivakov</i>	322	60.5	305	54.7
<i>Ruggieri, il Per</i>	324	61.2	305	54.7
Average	314	60.5	305	57.5

*Shading indicates estimated values.

How do we get from these estimated tap tones and weights to the radiation ratio of the wood? From practical experience in trying to copy the *Booth Stradivari*, I have found that it takes wood with a measured radiation ratio of about 16.7 along the grain to match mode 5 *and* the weight. Since radiation ratio works linearly, i.e., doubling the radiation ratio will halve the weight of the top, for a given tap tone frequency. In Table 5 I give estimated values of the radiation ratio for the five tops for which I had direct (rather than estimated) measurements of the plate without a bass bar. For each top, I multiplied 16.7 by 1+ the fractional change in weight. Note that, in doing so, I have ignored any influence of the radiation ratio across the grain. I have also assumed that all tops had the same geometry, which was clearly not the case.

Table 5. Estimated radiation ratios for five Old Italian violin tops.

Instrument	M 5 (Hz)	Wt. (g)	Radiation Ratio
<i>Booth Stradivari</i> , 1716	305	54	16.7
<i>Kreutzer Stradivari</i> , 1727	305	65.1	13.9
<i>Petri Stradivari</i> , 1700	305	56	16.1
<i>Stretton Guarneri</i> , 1726	305	63.8	14.1
<i>Testore, Spivakov</i>	305	54.7	16.5
Average	305	57.5	15.3

The values listed in Table 5 are at least a first approximation of the range of material properties found in Old Italian violin tops. The lowest value of 13.9 for a radiation ratio is well within the range of normally available wood. The average value of 15.3 is harder to find and for European spruce is typically associated with densities below 0.4 gm/cm³. Looking at the values for the *Booth Stradivari* and the *Testore* violins, spruce with a radiation ratio of 16.5–16.7 is very difficult to find indeed. These estimates correspond reasonably well with my own experience in trying to

copy specific tops. I have found that, once wood with the required radiation ratio along the grain is selected, that the value across the grain will tend to be too high. As a result, if you match mode 5 of the top you are copying, mode 2 will tend to be too high.

Summary Comment and Request

A larger database for violin tops, both new and old, would go a long way to providing more reliable statistics and would doubtlessly introduce new perspectives. If readers with tap tone and weight measurements for either new or old instruments would consider sharing their data, please contact this author at the address listed at the top of this article.

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